

16.6 Videos Guide

16.6a

- Description of and notation for parametric surfaces
 - $\mathbf{r}(u, v) = x(u, v)\mathbf{i} + y(u, v)\mathbf{j} + z(u, v)\mathbf{k}$
 - Parametric equations are $x = x(u, v)$, $y = y(u, v)$, $z = z(u, v)$
- Planes as parametric surfaces
 - A plane that contains a point P_0 , which corresponds to $\mathbf{r}_0 = \langle x_0, y_0, z_0 \rangle$, and vectors \mathbf{a} and \mathbf{b} has equation $\mathbf{r}(u, v) = \mathbf{r}_0 + u\mathbf{a} + v\mathbf{b}$
- Spheres as parametric surfaces
 - A sphere of radius a and center $(0, 0, 0)$ is parameterized as $\mathbf{r}(\phi, \theta) = a \cos \theta \sin \phi \mathbf{i} + a \sin \theta \sin \phi \mathbf{j} + a \cos \phi \mathbf{k}$, $0 \leq \phi \leq \pi$, $0 \leq \theta \leq 2\pi$
- Functions as parametric surfaces
 - A surface given by a function $z = f(x, y)$ is parameterized as $\mathbf{r}(x, y) = x \mathbf{i} + y \mathbf{j} + f(x, y) \mathbf{k}$

Exercises:

16.6b

- Identify the surface with the given vector equation.
 - $\mathbf{r}(u, v) = u^2 \mathbf{i} + u \cos v \mathbf{j} + u \sin v \mathbf{k}$
 - $\mathbf{r}(s, t) = \langle 3 \cos t, s, \sin t \rangle$, $-1 \leq s \leq 1$

16.6c

- Find a parametric representation for the surface.
 - The part of the part of the cylinder $x^2 + z^2 = 9$ that lies above the xy -plane and between the planes $y = -4$ and $y = 4$.
 - The part of the plane $z = x + 3$ that lies inside the cylinder $x^2 + y^2 = 1$.

16.6d

- Tangent planes
 - The tangent plane to a surface given by $\mathbf{r}(u, v)$ has normal vector $\mathbf{r}_u \times \mathbf{r}_v$

Exercise:

- Find an equation of the tangent plane to the given parametric surface at the specified point.
 $x = u^2 + 1$, $y = v^3 + 1$, $z = u + v$; $(5, 2, 3)$

16.6e

- Surface area
 - If a smooth parametric surface S is given by $\mathbf{r}(u, v)$, $(u, v) \in D$ and S is covered exactly once as (u, v) covers D , then $A(S) = \iint_D |\mathbf{r}_u \times \mathbf{r}_v| dA$

- If $z = f(x, y)$, then $|\mathbf{r}_u \times \mathbf{r}_v| = \sqrt{1 + (f_x)^2 + (f_y)^2}$
- For a surface of revolution obtained by rotating $f(x)$, $a \leq x \leq b$ about the x -axis, use $x = x$, $y = f(x) \cos \theta$, $z = f(x) \sin \theta$, $0 \leq \theta \leq 2\pi$, so that
$$A(S) = \int_a^b \int_0^{2\pi} |\mathbf{r}_x \times \mathbf{r}_\theta| dA$$

16.6f

Exercise:

- Find the area of the part of the surface $z = 4 - 2x^2 + y$ that lies above the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 1)$.